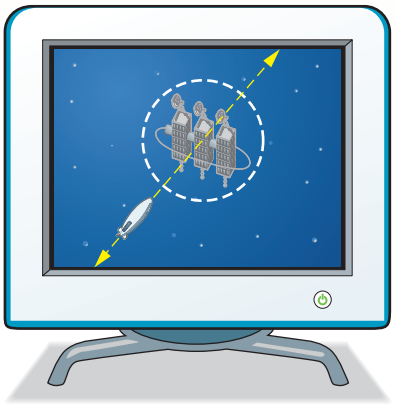
10-7 Solving Quadratic Systems

Main Ideas

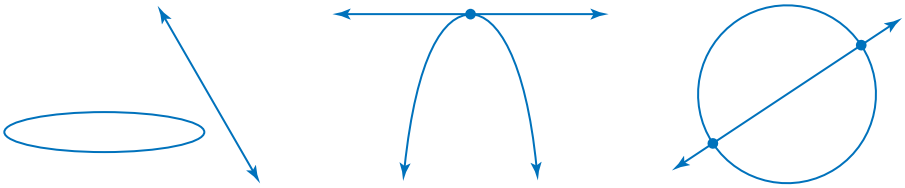
- Solve systems of quadratic equations algebraically and graphically.
- Solve systems of quadratic inequalities graphically.

GET READY for the Lesson

Suppose you are playing a computer game in which an enemy space station is located at the origin in a coordinate system. The space station is surrounded by a circular force field of radius 50 units. If the spaceship you control is flying toward the center along the line with equation $y = 3x$, the point where the ship hits the force field is a solution of a system of equations.



Systems of Quadratic Equations If the graphs of a system of equations are a conic section and a line, the system may have zero, one, or two solutions. Some of the possible situations are shown below.



no solutions | one solution | two solutions

You have solved systems of linear equations graphically and algebraically. You can use similar methods to solve systems involving quadratic equations.

Study Tip

Look Back

To review **solving systems of linear equations**, see Lesson 3-2.

EXAMPLE Linear-Quadratic System

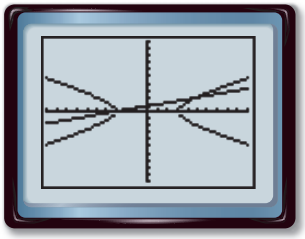
1 **Solve the system of equations.**

$x^2 - 4y^2 = 9$

$4y - x = 3$

You can use a graphing calculator to help visualize the relationships of the graphs of the equations and predict the number of solutions.

Solve each equation for y to obtain $y = \pm\frac{\sqrt{x^2 - 9}}{2}$ and $y = \frac{1}{4}x + \frac{3}{4}$. Enter $y = \frac{\sqrt{x^2 - 9}}{2}$, $y = -\frac{\sqrt{x^2 - 9}}{2}$, and $y = \frac{1}{4}x + \frac{3}{4}$ on the Y= screen. The graph indicates that the hyperbola and line intersect in two points. So the system has two solutions.



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Use substitution to solve the system. First rewrite $4y - x = 3$ as $x = 4y - 3$.

$$x^2 - 4y^2 = 9 \qquad \text{First equation in the system}$$
$$(4y - 3)^2 - 4y^2 = 9 \qquad \text{Substitute } 4y - 3 \text{ for } x.$$
$$12y^2 - 24y = 0 \qquad \text{Simplify.}$$
$$y^2 - 2y = 0 \qquad \text{Divide each side by 12.}$$
$$y(y - 2) = 0 \qquad \text{Factor.}$$
$$y = 0 \quad \text{or} \quad y - 2 = 0 \qquad \text{Zero Product Property}$$
$$y = 2 \qquad \text{Solve for } y.$$

Now solve for x.

$x = 4y - 3$	Equation for x in terms of y	$x = 4y - 3$
$= 4(0) - 3$	Substitute the y-values.	$= 4(2) - 3$
$= -3$	Simplify.	$= 5$

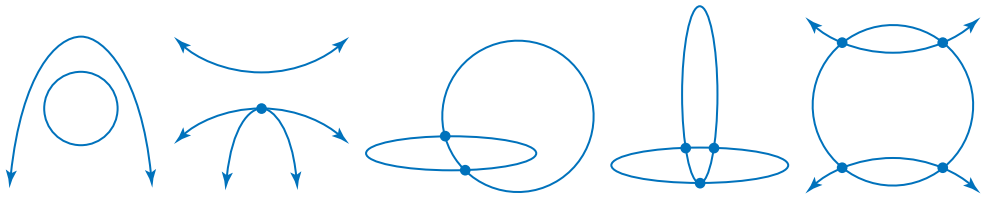
The solutions of the system are $(-3, 0)$ and $(5, 2)$. Based on the graph, these solutions are reasonable.

CHECK Your Progress **Solve each system of equations.**

1A. $y = x - 1$
$x^2 + y^2 = 25$

1B. $x + y = 1$
$y = x^2 - 5$

If the graphs of a system of equations are two conic sections, the system may have zero, one, two, three, or four solutions. Here are possible situations.



no solutions | one solution | two solutions | three solutions | four solutions

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Animation
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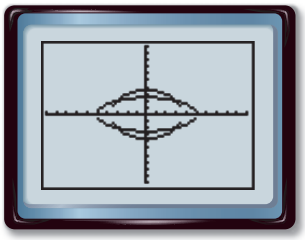
EXAMPLE **Quadratic-Quadratic System**

2 **Solve the system of equations.**

$$y^2 = 13 - x^2$$
$$x^2 + 4y^2 = 25$$

A graph of the system indicates that the circle and ellipse intersect in four points. So, this system has four solutions. Use the elimination method to solve.



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$$-x^2 - y^2 = -13 \qquad \text{Rewrite the first original equation.}$$
$$(+)\ x^2 + 4y^2 = 25 \qquad \text{Second original equation}$$
$$3y^2 = 12 \qquad \text{Add.}$$
$$y^2 = 4 \qquad \text{Divide each side by 3.}$$
$$y = \pm 2 \qquad \text{Take the square root of each side.}$$

Study Tip
Graphing Calculators
If you use ZSquare on the ZOOM menu, the graph of the first equation will look like a circle.

Substitute 2 and -2 for y in either of the original equations and solve for x.

$x^2 + 4y^2 = 25$	Second original equation	$x^2 + 4y^2 = 25$
$x^2 + 4(2)^2 = 25$	Substitute for y.	$x^2 + 4(-2)^2 = 25$
$x^2 = 9$	Subtract 16 from each side.	$x^2 = 9$
$x = \pm 3$	Take the square root of each side.	$x = \pm 3$

The solutions are $(3, 2)$, $(-3, 2)$, $(-3, -2)$, and $(3, -2)$.

CHECK Your Progress **Solve each system of equations.**

2A. $x^2 + y^2 = 36$
$x^2 + 9y^2 = 36$

2B. $x^2 - y^2 = 8$
$x^2 + y^2 = 120$

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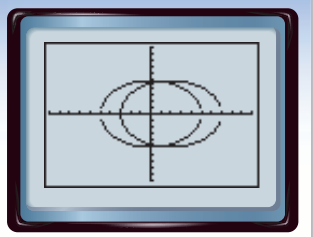
A graphing calculator can be used to approximate solutions of a system of quadratic equations.

GRAPHING CALCULATOR LAB

Quadratic Systems

THINK AND DISCUSS

The calculator screen shows the graphs of two circles.



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1. Write the system of equations represented.
2. Enter the equations into a TI-83/84 Plus and use the intersect feature on the CALC menu to solve the system. Round to the nearest hundredth.
3. Solve the system algebraically.
4. Can you always find the exact solution of a system using a graphing calculator? Explain.

Systems of Quadratic Inequalities Systems of quadratic inequalities are solved by graphing. As with linear inequalities, examine the inequality symbol to determine whether to include the boundary.

Study Tip

Graphing Quadratic Inequalities

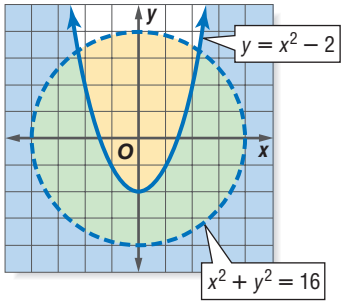
If you are unsure about which region to shade, you can test one or more points, as you did with linear inequalities.

EXAMPLE **System of Quadratic Inequalities**

3 **Solve the system of inequalities by graphing.**

$y \leq x^2 - 2$
$x^2 + y^2 < 16$

The intersection of the graphs, shaded green, represents the solution of the system.



CHECK $(0, -3)$ is in the shaded area. Use this point to check your solution.

$y \leq x^2 - 2$	$x^2 + y^2 < 16$
$-3 \leq (0)^2 - 2$	$0^2 + (-3)^2 < 16$
$-3 \leq -2$ true	$9 < 16$ true

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CHECK Your Progress

3. Solve by graphing. $x^2 + y^2 \leq 49$ and $y \geq x^2 + 1$

CHECK Your Understanding

Example 1 (pp. 603–604)

Find the exact solution(s) of each system of equations.

1. $y = 5$
$y^2 = x^2 + 9$

2. $y - x = 1$
$x^2 + y^2 = 25$

Example 2 (pp. 604–605)

3. $3x = 8y^2$
$8y^2 - 2x^2 = 16$

4. $5x^2 + y^2 = 30$
$9x^2 - y^2 = -16$

5. CELL PHONES A person using a cell phone can be located in respect to three cellular towers. In a coordinate system where a unit represents one mile, the caller is determined to be 50 miles from a tower at the origin, 40 miles from a tower at (0, 30), and 13 miles from a tower at (35, 18). Where is the caller?

Example 3 (pp. 605–606)

Solve each system of inequalities by graphing.

6. $x + y < 4$
$9x^2 - 4y^2 \geq 36$

7. $x^2 + y^2 < 25$
$4x^2 - 9y^2 < 36$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–13	1
14–19	2
20–25	3

Find the exact solution(s) of each system of equations.

8. $y = x + 2$
$y = x^2$

9. $y = x + 3$
$y = 2x^2$

10. $x^2 + y^2 = 36$
$y = x + 2$

11. $y^2 + x^2 = 9$
$y = 7 - x$

12. $\frac{x^2}{30} + \frac{y^2}{6} = 1$
$x = y$

13. $\frac{x^2}{36} - \frac{y^2}{4} = 1$
$x = y$

14. $4x + y^2 = 20$
$4x^2 + y^2 = 100$

15. $y + x^2 = 3$
$x^2 + 4y^2 = 36$

16. $x^2 + y^2 = 64$
$x^2 + 64y^2 = 64$

17. $y^2 + x^2 = 25$
$y^2 + 9x^2 = 25$

18. $y^2 = x^2 - 25$
$x^2 - y^2 = 7$

19. $y^2 = x^2 - 7$
$x^2 + y^2 = 25$

Solve each system of inequalities by graphing.

20. $x + 2y > 1$
$x^2 + y^2 \leq 25$

21. $x + y \leq 2$
$4x^2 - y^2 \geq 4$

22. $x^2 + y^2 \geq 4$
$4y^2 + 9x^2 \leq 36$

23. $x^2 + y^2 < 36$
$4x^2 + 9y^2 > 36$

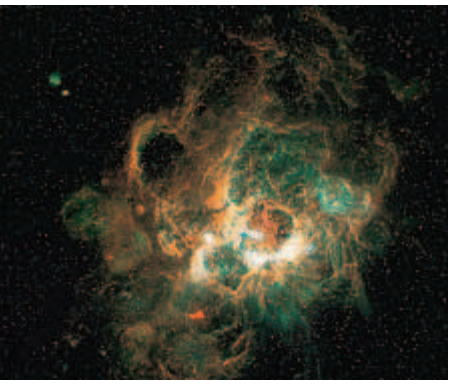
24. $y^2 < x$
$x^2 - 4y^2 < 16$

25. $x^2 \leq y$
$y^2 - x^2 \geq 4$

26. Graph each system of equations. Use the graph to solve the system.

a. $4x - 3y = 0$
$x^2 + y^2 = 25$

b. $y = 5 - x^2$
$y = 2x^2 + 2$



Real-World Link

The astronomical unit (AU) is the mean distance between Earth and the Sun. One AU is about 93 million miles or 150 million kilometers

Source: infoplease.com

ASTRONOMY **For Exercises 27 and 28, use the following information.**

The orbit of Pluto can be modeled by the equation $\frac{x^2}{39.5^2} + \frac{y^2}{38.3^2} = 1$, where the units are astronomical units. Suppose a comet is following a path modeled by the equation $x = y^2 + 20$.

27. Find the point(s) of intersection of the orbits of Pluto and the comet. Round to the nearest tenth.

28. Will the comet necessarily hit Pluto? Explain.

29. Where do the graphs of $y = 2x + 1$ and $2x^2 + y^2 = 11$ intersect?

30. What are the coordinates of the points that lie on the graphs of both $x^2 + y^2 = 25$ and $2x^2 + 3y^2 = 66$?

31. ROCKETS Two rockets are launched at the same time, but from different heights. The height y in feet of one rocket after t seconds is given by $y = -16t^2 + 150t + 5$. The height of the other rocket is given by $y = -16t^2 + 160t$. After how many seconds are the rockets at the same height?

32. ADVERTISING The corporate logo for an automobile manufacturer is shown at the right. Write a system of three equations to model this logo.

SATELLITES **For Exercises 33–35, use the following information.**

Two satellites are placed in orbit about Earth. The equations of the two orbits are $\frac{x^2}{(300)^2} + \frac{y^2}{(900)^2} = 1$ and $\frac{x^2}{(600)^2} + \frac{y^2}{(690)^2} = 1$, where distances are in kilometers and Earth is the center of each curve.

33. Solve each equation for y.

34. Use a graphing calculator to estimate the intersection points of the two orbits.

35. Compare the orbits of the two satellites.

Graphing Calculator

Write a system of equations that satisfies each condition. Use a graphing calculator to verify that you are correct.

36. two parabolas that intersect in two points

37. a hyperbola and a circle that intersect in three points

38. a circle and an ellipse that do not intersect

39. a circle and an ellipse that intersect in four points

40. a hyperbola and an ellipse that intersect in two points

41. two circles that intersect in three points

EXTRA PRACTICE
See pages 913, 935.
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H.O.T. Problems

42. REASONING Sketch a parabola and an ellipse that intersect in exactly three points.

43. OPEN ENDED Write a system of quadratic equations for which (2, 6) is a solution.

CHALLENGE **For Exercises 44–48, find all values of k for which the system of equations has the given number of solutions. If no values of k meet the condition, write *none*.**

$$x^2 + y^2 = k^2 \qquad \frac{x^2}{9} + \frac{y^2}{4} = 1$$

44. no solutions **45.** one solution **46.** two solutions

47. three solutions **48.** four solutions

49. Which One Doesn't Belong? Which system of equations is NOT like the others? Explain your reasoning.

$x^2 + y^2 = 16$
$x + y = 3$

$x^2 + y^2 = 25$
$\frac{x^2}{25} - \frac{y^2}{16} = 1$

$x^2 + y^2 = 20$
$\frac{x^2}{25} + \frac{y^2}{16} = 1$

$y - 2x = -5$
$y^2 + x = 9$

50. *Writing in Math* Use the information on page 603 to explain how systems of equations apply to video games. Include a linear-quadratic system of equations that applies to this situation and the coordinates of the point at which the spaceship will hit the force field, assuming that the spaceship moves from the bottom of the screen toward the center.

STANDARDIZED TEST PRACTICE

51. ACT/SAT How many solutions does the system of equations $\frac{x^2}{5^2} - \frac{y^2}{3^2} = 1$ and $(x - 3)^2 + y^2 = 9$ have?

A 0

B 1

C 2

D 4

52. REVIEW Given: Two angles are supplementary. One angle is 25° more than the measure of the other angle. Conclusion: The measures of the angles are 65° and 90°. This conclusion—

F is contradicted by the first statement given.

G is verified by the first statement given.

H invalidates itself because a 90° angle cannot be supplementary to another.

J verifies itself because 90° is 25° more than 65°.

Spiral Review

Write each equation in standard form. State whether the graph of the equation is a *parabola, circle, ellipse,* or *hyperbola*. Then graph the equation. (Lesson 10-6)

53. $x^2 + y^2 + 4x + 2y - 6 = 0$

54. $9x^2 + 4y^2 - 24y = 0$

55. Find the coordinates of the vertices and foci and the equations of the asymptotes of the hyperbola with the equation $6y^2 - 2x^2 = 24$. Then graph the hyperbola. (Lesson 10-5)

Cross-Curricular Project

Algebra and Earth Science

Earthquake Extravaganza It is time to complete your project. Use the information and data you have gathered about earthquakes to prepare a research report or Web page. Be sure to include graphs, tables, diagrams, and any calculations you need for the earthquake you chose.

MathOnline **Cross-Curricular Project at at algebra2.com**